CBSE Sample Question Paper–1 (2017-18)

Time: 3 Hours **Maximum Marks: 80**

General Instructions:

(*i*) All questions are **compulsory**.
(*ii*) The question paper consists of **30** questions divided into four sections—**A**, **B**, **C** and **D**.
(*iii*) Section **A** contains **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of **4** marks each.
(*iv*) There is no overall choice. However, an internal choice has been provided in four questions of **3** marks each and three questions of **4** marks each. You have to attempt only one of the alternatives in all such questions.
(*v*) Use of **calculators** is not permitted.

SECTION A

1. Write whether the rational number $\frac{7}{75}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion. **1**

Sol. $\frac{7}{75} = 0.0933333 ...... = 0.09\overline{3}$

So, it is a non-terminating repeating decimal expansion. 1

2. Find the value(s) of k, if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has equal roots. **1**

Sol. Given equation $3x^2 - k\sqrt{3}x + 4 = 0$ ...(1)

On comparing eqn. (1) with $ax^2 + bx + c = 0$, we get

$$a = 3, \quad b = -k\sqrt{3}, \quad c = 4,$$

For equal roots, we have $b^2 - 4ac = 0$ *i.e.* $b^2 = 4ac$

$\therefore$ On using $b^2 = 4ac$, we get

$$(-k\sqrt{3})^2 = 4 \times 3 \times 4$$

$$3k^2 = 4 \times 3 \times 4 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$$

3. Find the eleventh term from the last term of the AP:
27, 23, 19, ..., –65. **1**

Sol. Given series: 27, 23, 19,..., – 65

Here, $a = 27$, $d = 23 - 27 = -4$, $l = -65$

nth term from the end (last) $= l - (n - 1)d$

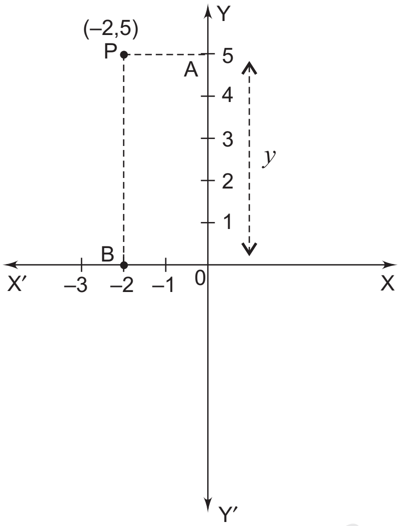
$\therefore$ 11th term from the end (last) $= l - (11 - 1)d$

$$= l - 10d = -65 - 10(-4)$$

$$= -65 + 40 = -25$$ 1

4. Find the coordinates of the point on y-axis which is nearest to the point (– 2, 5). **1**

Sol.



Let Co-ordinate of point A on the Y-axis is $(0, y)$ or (0, OA)

From the graph, it is clear that OA = $y = 5$

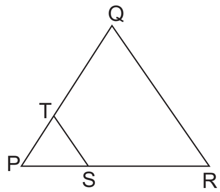
$\therefore$ Co-ordinate of point A is (0, 5) 1

5. Given PS = 3 cm, SR = 4 cm. From given figure, we have

$$PR = PS + SR = 3 \text{ cm} + 4 \text{ cm} = 7 \text{ cm}$$

$$\frac{ar(\triangle PST)}{ar(\triangle PRQ)} = \frac{PS^2}{PR^2} \quad \text{[using Area-ratio theorem]}$$

$$\therefore \quad \frac{ar(\triangle PST)}{ar(\triangle PRQ)} = \frac{(3)^2}{(7)^2} = \frac{9}{49} \qquad 1$$



6. If $\cos A = \frac{2}{5}$, find the value of $4 + 4\tan^2 A$ **1**

Sol. Given: $\cos A = \frac{2}{5}$, then $\sec A = \frac{1}{\cos A} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$

$$4 + 4\tan^2 A = 4(1 + \tan^2 A) = 4\sec^2 A = 4 \times \left(\frac{5}{2}\right)^2 = 4 \times \frac{25}{4} = 25 \quad 1$$

SECTION B

7. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$; a, b are prime numbers, then verify: LCM (p, q) × HCF $(p, q) = pq$ **2**

Sol. LCM $(p, q) = a^3b^3$ and HCF $(p, q) = a^2b$ ½

LCM (p, q) × HCF $(p, q) = a^5b^4 = (a^2b^3)(a^3 b) = pq$ 1

8. The sum of first n terms of an AP is given by $S_n = 2n^2 + 3n$. Find the sixteenth term of the AP. **2**

Sol.

$S_n = 2n^2 + 3n$ ½

$S_1 = 5 = a_1$ ½

$S_2 = a_1 + a_2 = 14 \Rightarrow a_2 = 9$ ½

$d = a_2 - a_1 = 4$

$a_{16} = a_1 + 15d = 5 + 15(4) = 65$ ½

9. Find the value(s) of k for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions. **2**

Sol. For pair of equations $kx + 1y = k^2$ and $1x + ky = 1$

We have: $\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ½

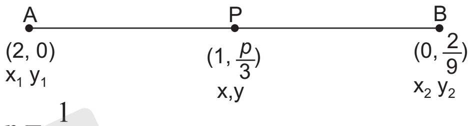
$\therefore \quad \frac{k}{1} = \frac{1}{k} \Rightarrow k^2 = 1 \Rightarrow k = 1, -1$ ...(i) ½

and $\frac{1}{k} = \frac{k^2}{1} \Rightarrow k^3 = 1 \Rightarrow k = 1$ ...(ii) ½

From (i) and (ii), $k = 1$ ½

10. If $\left(1, \frac{p}{3}\right)$ is the mid-point of the line segment joining the points (2, 0) and $\left(0, \frac{2}{9}\right)$, then show that the line $5x + 3y + 2 = 0$ passes through the point $(-1, 3p)$. **2**

Sol. We know that $y = \frac{y_1 + y_2}{2}$



$$\frac{p}{3} = \frac{0 + \frac{2}{9}}{2} \Rightarrow p = \frac{1}{3}$$

Now, co-ordinate of point $(-1, 3p)$ is equal to $\left(-1, 3\times\frac{1}{3}\right)$ *i.e.* $(-1, 1)$.

The given line $5x + 3y + 2 = 0$ passes through the point $(-1, 1)$ as $5(-1) + 3(1) + 2 = -5 + 5 = 0$

11. A box contains cards numbered 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is **2**

(i) a square number (ii) a multiple of 7

Sol. Total number of cards in the box are 113.

(i) Cards having square number of them between 11 and 123 are 16, 25, 36, 49, 64, 81, 100, 121; so there are total eight (8) square numbers.

$\therefore \quad$ P (square number) $= \frac{8}{113}$ 1

(ii) Cards having a number, which is multiple of 7 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119. So, there are total sixteen (16) number, which are multiple of 7.

$\therefore \quad$ P (multiple of 7) $= \frac{16}{113}$ 1

12. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the bag. **2**

Sol. Let number of red balls be x

$\therefore \quad$ P(red ball) $= \frac{x}{12}$ **½**

If 6 more red balls are added:

The number of red balls $= x + 6$

P(red ball) $= \frac{x+6}{18}$

Since, $\frac{x+6}{18} = 2\left(\frac{x}{12}\right) \Rightarrow x = 3$ 1

$\therefore$ There are 3 red balls in the bag. **½**

SECTION C

13. Show that exactly one of the numbers n, $n + 2$ or $n + 4$ is divisible by 3. **3**

Sol. Let $n = 3k, 3k + 1$ or $3k + 2$.

(i) When $n = 3k$:

n is divisible by 3. $\quad n + 2 = 3k + 2 \Rightarrow n + 2$ is not divisible by 3.

$$n + 4 = 3k + 4 = 3(k + 1) + 1$$ 1

$\Rightarrow$ $n + 4$ is not divisible by 3.

(ii) When $n = 3k + 1$:

n is not divisible by 3. $\quad n + 2 = (3k + 1) + 2 = 3k + 3 = 3(k + 1)$

$\Rightarrow$ $n + 2$ is divisible by 3.

$$n + 4 = (3k + 1) + 4 = 3k + 5 = 3(k + 1) + 2$$ 1

$\Rightarrow$ $n + 4$ is not divisible by 3.

(iii) When $n = 3k + 2$:

n is not divisible by 3.

$$n + 2 = (3k + 2) + 2 = 3k + 4 = 3(k + 1) + 1$$

$\Rightarrow$ $n + 2$ is not divisible by 3.

$$n + 4 = (3k + 2) + 4 = 3k + 6 = 3(k + 2)$$

$\Rightarrow$ $n + 4$ is divisible by 3. 1

Hence, exactly one of the numbers n, $n + 2$ or $n + 4$ is divisible by 3.

14. Find all the zeroes of the polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$ if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$. **3**

Sol. Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the two zeroes therefore, $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2 - 5)$ 1

is a factor of given polynomial.

We divide the given polynomial by $3x^2 - 5$.

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline 3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & \pm 3x^4 \qquad \mp 5x^2 \\ \hline & 6x^3 + 3x^2 - 10x - 5 \\ & \pm 6x^4 \qquad \mp 10x \\ \hline & 3x^2 - 5 \\ & \pm 3x^2 \mp 5 \\ \hline & 0 \end{array}$$ 1

For other zeroes, $x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0, x = -1, -1$

$\therefore$ Zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1. 1

15. Seven times a two-digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number. **3**

Sol. Let the tens and the units digit be y and x respectively.

So, the number is $10y + x$. ½

The number when digits are reversed is $10x + y$. ½

Now, $\quad 7(10y + x) = 4(10x + y) \Rightarrow 2y = x \quad \ldots(i)$ 1

Also $\quad x - y = 3 \quad \ldots(ii)$ ½

Solving (1) and (2), we get $y = 3$ and $x = 6$.

Hence, the number is 36. ½

16. In what ratio does the x-axis divide the line segment joining the points (– 4, – 6) and (– 1, 7)? Find the coordinates of the point of division. **3**

OR

The points A(4, – 2), B(7, 2), C(0, 9) and D(– 3, 5) form a parallelogram. Find the length of the altitude of the parallelogram on the base AB.

Sol. Let x-axis divides the line segment joining (– 4, – 6) and (– 1, 7) at the point P in the ratio 1 : k. ½

Now, coordinates of point of division P $\left(\frac{-1-4k}{k+1}, \frac{7-6k}{k+1}\right)$

Since P lies on x-axis, therefore $\frac{7-6k}{k+1} = 0$ 1

$\Rightarrow \quad 7 - 6k = 0 \quad \Rightarrow \quad k = \frac{7}{6}$

Hence the ratio is $1 : \frac{7}{6} = 6 : 7$ ½

Now, the coordinates of P are $\left(\frac{-1-4\left(\frac{7}{6}\right)}{\frac{7}{6}+1}, \frac{7-6\left(\frac{7}{6}\right)}{\frac{7}{6}+1}\right)$ *i.e.* $\left(\frac{\frac{-17}{3}}{\frac{13}{6}}, 0\right) = \left(\frac{-34}{13}, 0\right)$. 1

OR

Let the height of parallelogram taking AB as base be h.

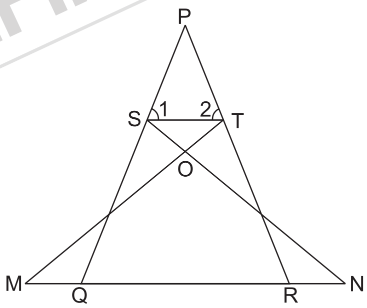
Now $\quad AB = \sqrt{(7-4)^2 + (2+2)^2} = \sqrt{3^2 + 4^2} = 5$ units. 1

$\text{Area}(\Delta ABC) = \frac{1}{2}[4(2-9) + 7(9+2) + 0(-2-2)] = \frac{49}{2}$ sq. units. 1

Now, $\quad \frac{1}{2} \times AB \times h = \frac{49}{2} \quad \Rightarrow \quad \frac{1}{2} \times 5 \times h = \frac{49}{2}$

$\Rightarrow \quad h = \frac{49}{5} = 9.8$ units. 1

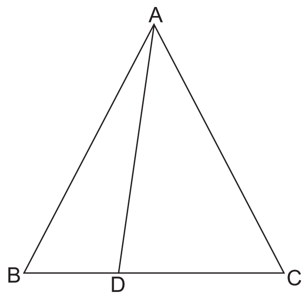
17. In the given figure, ∠1 = ∠2 and ΔNSQ ≅ ΔMTR, then prove that ΔPTS ~ ΔPRQ. **3**



OR

In an equilateral triangle ABC, D is a point on the side BC such that

$BD = \frac{1}{3} BC$. Prove that $9AD^2 = 7AB^2$



Sol. ∠SQN = ∠TRM

(CPCT as ΔNSQ ≅ ΔMTR) 1

Since, ∠P + ∠1 + ∠2 = ∠P + ∠PQR + ∠PRQ

(Angle sum property)

$\Rightarrow$ ∠1 + ∠2 = ∠PQR + ∠PRQ

$\Rightarrow \quad 2\angle 1 = 2\angle PQR$

(as $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$) **1**

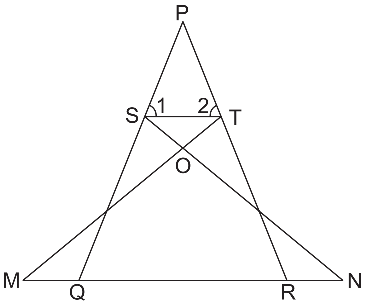
$\angle 1 = \angle PQR$

Also $\quad \angle 2 = \angle PRQ$

And $\quad \angle SPT = \angle QPR \quad$ (Common)

$\therefore \quad \Delta PTS \sim \Delta PRQ$

(By AAA similarity criterion) **1**



OR

Construction: Draw $AP \perp BC$ ½

In ΔADP, $\quad AD^2 = AP^2 + DP^2$ ½

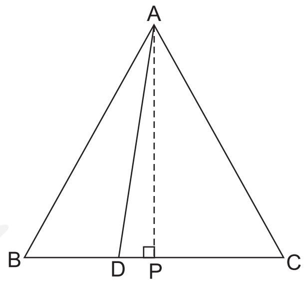
$AD^2 = AP^2 + (BP - BD)^2$ ½

$AD^2 = AP^2 + BP^2 + BD^2 - 2(BP)(BD)$

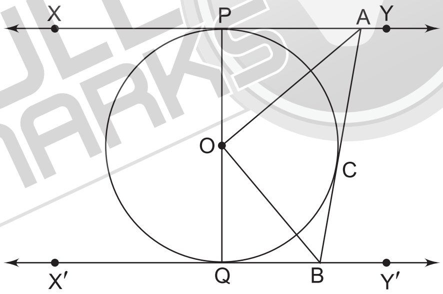
$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$ 1

$AD^2 = \frac{7}{9}AB^2 \quad (\because \; BC = AB)$

$9AD^2 = 7AB^2$ ½



18. In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$. **3**



Sol. Join OC

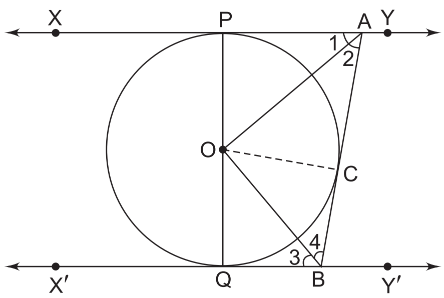
In ΔOPA and ΔOCA

$OP = OC$ (Radii of same circle)

$PA = CA$ (Length of two tangents)

$AO = AO$ (Common) **1**

$\therefore \quad \Delta OPA \cong \Delta OCA$ (By SSS congruency criterion)



Hence, $\quad \angle 1 = \angle 2$ (CPCT) **1**

Similarly, $\quad \angle 3 = \angle 4$

Now, $\quad \angle PAB + \angle QBA = 180°$

$\Rightarrow \quad 2\angle 2 + 2\angle 4 = 180° \quad \Rightarrow \quad \angle 2 + \angle 4 = 90°$

$\Rightarrow \quad \angle AOB = 90°$ (Angle sum property)

19. Evaluate: $\dfrac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°} + \dfrac{\sin^2 63° + \cos 63° \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}$ **3**

OR

If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate: $\tan\theta + \cot\theta$

Sol. $\dfrac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°} + \dfrac{\sin^2 63° + \cos 63° \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}$

$= \dfrac{\text{cosec}^2 63° + \tan^2 24°}{\tan^2(90° - 66°) + \text{cosec}^2(90° - 27°)}$

$+ \dfrac{\sin^2 63° + \cos 63° \cos(90° - 27°) + \sin 27° \text{cosec}(90° - 63°)}{2[\text{cosec}^2 65° - \cot^2(90° - 25°)]}$ 1

$= \dfrac{\text{cosec}^2 63° + \tan^2 24°}{\tan^2 24° + \text{cosec}^2 63°} + \dfrac{\sin^2 63° + \cos^2 63 + \sin 27° \text{cosec } 27°}{2(\text{cosec}^2 65° - \cot^2 65°)} = 1 + \dfrac{1+1}{2(1)} = 2$ 1 + 1

OR

$\sin\theta + \cos\theta = \sqrt{2}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = (\sqrt{2})^2 \Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$ ½

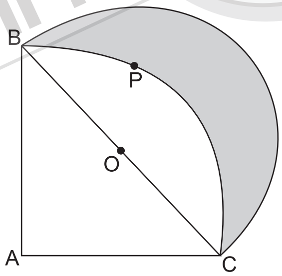
$\Rightarrow 1 + 2\sin\theta\cos\theta = 2 \Rightarrow \sin\theta\cos\theta = \dfrac{1}{2}$ ...(*i*) 1

We know, $\sin^2\theta + \cos^2\theta = 1$ ...(*ii*) ½

Dividing (*ii*) by (*i*), we get

$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{1/2} \Rightarrow \tan\theta + \cot\theta = 2$ 1

20. In the given figure, ABPC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. **3**

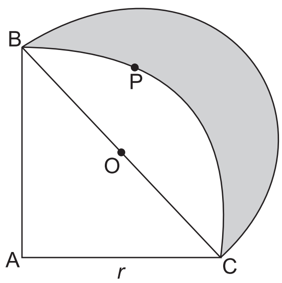


Sol. We know, $AC = r$

In ΔACB, $BC^2 = AC^2 + AB^2$

$\Rightarrow BC = AC\sqrt{2}$ ($\because$ AB = AC)

$\Rightarrow BC = r\sqrt{2}$ 1



Required area = *ar*(ΔACB) + *ar*(semicircle on BC as diameter) – *ar*(quadrant ABPC)

$= \dfrac{1}{2} \times r \times r + \dfrac{1}{2} \times \pi \times \left(\dfrac{r\sqrt{2}}{2}\right)^2 - \dfrac{1}{4}\pi r^2$

$= \dfrac{r^2}{2} + \dfrac{\pi r^2}{4} - \dfrac{\pi r^2}{4} = \dfrac{r^2}{2} = \dfrac{196}{2} \text{ cm}^2 = 98 \text{ cm}^2$ 1+1

21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed? **3**

OR

A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the remaining solid after the cone is carved out.

Sol. Let the area that can be irrigated in 30 minutes be A m^2.

Water flowing in canal in 30 minutes = $\left(10,000 \times \frac{1}{2}\right)$ m = 5000 m ½

Volume of water flowing out in 30 minutes = $(5000 \times 6 \times 1.5)\ m^3 = 45000\ m^3$ …(*i*) 1

Volume of water required to irrigate the field = A × $\frac{8}{100}$ m^3 ...(*ii*) ½

Equating (*i*) and (*ii*), we get

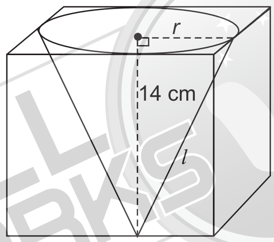
$$A \times \frac{8}{100} = 45000 \quad \Rightarrow \quad A = 562500\ m^2.$$ 1

OR

Given: Side of cube (a) = 14 cm

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 14^2} = 7\sqrt{5} \quad \text{and} \quad r = \frac{\text{side}}{2} = \frac{14}{2}\ \text{cm} = 7\ \text{cm}$$ ½

Surface area of remaining solid = $6a^2 - \pi r^2 + \pi rl$, where r and l are the radius and slant height of the cone. 1



$$= 6 \times 14 \times 14 - \frac{22}{7} \times 7 \times 7 + \frac{22}{7} \times 7 \times 7\sqrt{5}$$ 1

$$= 1176 - 154 + 154\sqrt{5} = (1022 + 154\sqrt{5})\ cm^2$$ ½

22. Find the mode of the following distribution of marks obtained by the students in an examination:

Marks obtained	0-20	20-40	40-60	60-80	80-100
Number of students	15	18	21	29	17

Given the mean of the above distribution is 53, using empirical relationship estimate the value of its median. **3**

Sol.

$$\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$ 1

$$= 60 + \left(\frac{29 - 21}{58 - 21 - 17}\right) \times 20 = 68$$ 1

So, the mode marks is 68.

Empirical relationship between the three measures of central tendencies is:

3 Median = Mode + 2 Mean

3 Median = 68 + 2 × 53 1

Median = 58 marks

SECTION D

23. A train travelling at a uniform speed for 360 km would have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train. **4**

OR

Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots and if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.

Sol. Let original speed of the train be x km/h.

Time taken at original speed = $\frac{360}{x}$ hours 1

Time taken at increased speed = $\frac{360}{x+5}$ hours ½

Now, $\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$ 1½

$\Rightarrow$ $360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5} \Rightarrow x^2 + 5x - 2250 = 0$

$\Rightarrow$ $x = 45$ or -50 (as speed cannot be negative)

$\Rightarrow$ $x = 45$ km/h 1

OR

Discriminant = $b^2 - 4ac = 36 - 4 \times 5 \times (-2) = 76 > 0$ 1

So, the given equation has two distinct real roots

$$5x^2 - 6x - 2 = 0$$

Multiplying both sides by 5.

$$(5x)^2 - 2 \times (5x) \times 3 = 10 \Rightarrow (5x)^2 - 2 \times (5x) \times 3 + 3^2 = 10 + 3^2$$

$\Rightarrow$ $(5x-3)^2 = 19 \Rightarrow 5x - 3 = \pm\sqrt{19}$ 1

$$x = \frac{3 \pm \sqrt{19}}{5}$$ 1

Verification:

$5\left(\frac{3+\sqrt{19}}{5}\right)^2 - 6\left(\frac{3+\sqrt{19}}{5}\right) - 2 = \frac{9 + 6\sqrt{19} + 19}{5} - \frac{18 + 6\sqrt{19}}{5} - \frac{10}{5} = 0$ ½

Similarly, $5\left(\frac{3-\sqrt{19}}{5}\right)^2 - 6\left(\frac{3-\sqrt{19}}{5}\right) - 2 = 0$ ½

24. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the AP. **4**

Sol. Let the three middle most terms of the AP be $a - d, a, a + d$.

We have, $(a - d) + a + (a + d) = 225$ 1

$\Rightarrow$ $3a = 225 \Rightarrow a = 75$ ½

Now, the AP is

$a - 18d, \ldots, a - 2d, a - d, a, a + d, a + 2d, \ldots, a + 18d$

Sum of last three terms:

$(a + 18d) + (a + 17d) + (a + 16d) = 429$ 1

$\Rightarrow$ $3a + 51d = 429 \Rightarrow a + 17d = 143$

$\Rightarrow$ $75 + 17d = 143 \Rightarrow d = 4$ ½

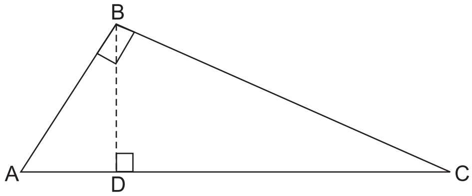
Now, first term = $a - 18d = 75 - 18(4) = 3$

$\therefore$ The AP is 3, 7, 11, …, 147. 1

25. Show that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. **4**

OR

Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



Sol. **Given:** A right triangle ABC right angled at B.

To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$ ½

Proof: In ΔADB and ΔABC

$\angle ADB = \angle ABC$ (Each 90°)

$\angle BAD = \angle CAB$ (Common)

$\therefore$ $\Delta ADB \sim \Delta ABC$ (By AA similarity criterion) 1

Now, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$ (Corresponding sides are proportional) 1

$\Rightarrow$ $AB^2 = AD \times AC$ …(*i*) 1

Similarly, $\Delta BDC \sim \Delta ABC$

$\Rightarrow$ $BC^2 = CD \times AC$ …(*ii*) ½

Adding (*i*) and (*ii*), $AB^2 + BC^2 = AD \times AC + CD \times AC$

$\Rightarrow$ $AB^2 + BC^2 = AC \times (AD + CD)$

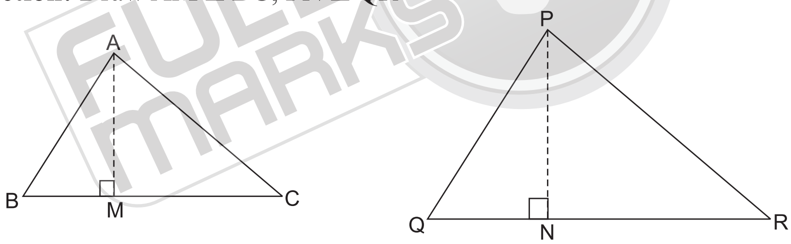
$\Rightarrow$ $AB^2 + BC^2 = AC^2$, Hence Proved. 1

OR

Given: $\Delta ABC \sim \Delta PQR$

To prove: $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Construction: Draw $AM \perp BC$, $PN \perp QR$ ½



$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC}{QR} \times \frac{AM}{PN} \quad ...(i) \quad 1$$

In ΔABM and ΔPQN, $\angle B = \angle Q$ ($\because$ $\Delta ABC \sim \Delta PQR$)

$\angle M = \angle N$ (Each 90°)

$\Delta ABM \sim \Delta PQN$ (AA similarity criterion)

Therefore, $\dfrac{AM}{PN} = \dfrac{AB}{PQ}$ …(*ii*) 1

But $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$ ($\because$ $\Delta ABC \sim \Delta PQR$) …(*iii*) ½

Hence, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC}{QR} \times \dfrac{AM}{PN}$ [From (*i*)]

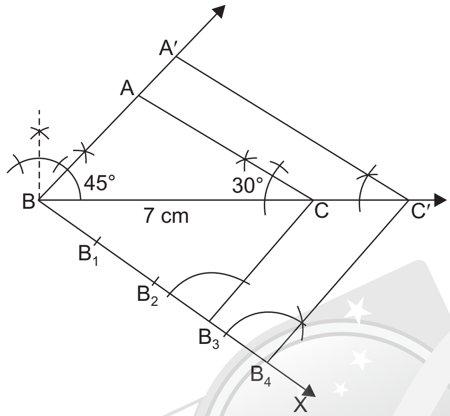
$= \dfrac{AB}{PQ} \times \dfrac{AB}{PQ} = \left(\dfrac{AB}{PQ}\right)^2$ [From (*ii*) and (*iii*)] ½

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$ [Using (*iii*)] ½

26. Draw a triangle ABC with side BC = 7 cm, ∠B = 45°, ∠A = 105°. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC. **3**

Sol. Draw ΔABC in which BC = 7 cm, ∠B = 45°, ∠A = 105° and hence ∠C = 180° – (45° + 105°) = 30°. 1

Construction of similar triangle A′ BC′ as shown below: 3



27. Prove that $\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$ **4**

Sol.

$$\text{LHS} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} \times \frac{\cos\theta + \sin\theta + 1}{\cos\theta + \sin\theta + 1}$$ 1

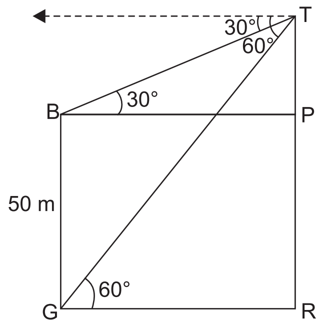
$$= \frac{(\cos\theta + 1)^2 - \sin^2\theta}{(\cos\theta + \sin\theta)^2 - 1^2} = \frac{\cos^2\theta + 1 + 2\cos\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1}$$ 1

$$= \frac{2\cos^2\theta + 2\cos\theta}{2\sin\theta\cos\theta}$$ 1

$$= \frac{2\cos\theta(\cos\theta + 1)}{2\sin\theta\cos\theta} = \frac{\cos\theta + 1}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta = \text{RHS}$$ 1

28. The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60°, respectively. Find the height of the tower and also the horizontal distance between the building and the tower. **4**

Sol.



Correct Figure 1

In ΔBTP ⇒ $\tan 30° = \frac{TP}{BP}$ ⇒ $\frac{1}{\sqrt{3}} = \frac{TP}{BP}$

$$BP = TP\sqrt{3} \quad ...(i) \qquad ½$$

In ΔGTR, $\tan 60° = \frac{TR}{GR} \Rightarrow \sqrt{3} = \frac{TR}{GR} \Rightarrow GR = \frac{TR}{\sqrt{3}}$ ...(*ii*) ½

Now, $TP\sqrt{3} = \frac{TR}{\sqrt{3}}$ (as BP = GR)

$\Rightarrow$ $3TP = TP + PR$

$\Rightarrow$ $2TP = BG \Rightarrow TP = \frac{50}{2}\text{ m} = 25\text{ m}$ 1

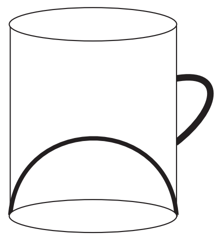
Now, $TR = TP + PR = (25 + 50)$ m.

Height of tower = TR = 75 m. **½**

Distance between building and tower = GR = $\frac{TR}{\sqrt{3}}$

$\Rightarrow$ $GR = \frac{75}{\sqrt{3}}\text{ m} = 25\sqrt{3}\text{ m}$ **½**

29. Two dairy owners A and B sell flavoured milk filled to capacity in mugs of negligible thickness, which are cylindrical in shape with a raised hemispherical bottom. The mugs are 14 cm high and have diameter of 7 cm as shown in given figure. Both A and B sell flavoured milk at the rate of ₹ 80 per litre. The dairy owner A uses the formula $\pi r^2 h$ to find the volume of milk in the mug and charges ₹ 43.12 for it. The dairy owner B is of the view that the price of actual quantity of milk should be charged. What according to him should be the price of one mug of milk? Which value is exhibited by the dairy owner B? $\left(\text{use } \pi = \frac{22}{7}\right)$ **4**



Sol. Capacity of mug (actual quantity of milk) = $\pi r^2 h - \frac{2}{3}\pi r^3$ 1

$$= \pi r^2\left(h - \frac{2}{3}r\right) = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(14 - \frac{2}{3} \times \frac{7}{2}\right)$$

$$= \frac{2695}{6}\text{ cm}^3 \qquad 1$$

Amount dairy owner B should charge for one mug of milk

$$= \frac{2695}{6} \times \frac{80}{1000} = ₹\ 35.93 \qquad 1$$

Value exhibited by dairy owner B: honesty (or any similar value) 1

30. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency k. **4**

Daily pocket allowance (in ₹)	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Number of children	3	6	9	13	k	5	4

OR

The following frequency distribution shows the distance (in metres) thrown by 68 students in a Javelin throw competition.

Distance (in m)	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Number of students	4	5	13	20	14	8	4

Draw a less than type Ogive for the given data and find the median distance thrown using this curve.

Sol.

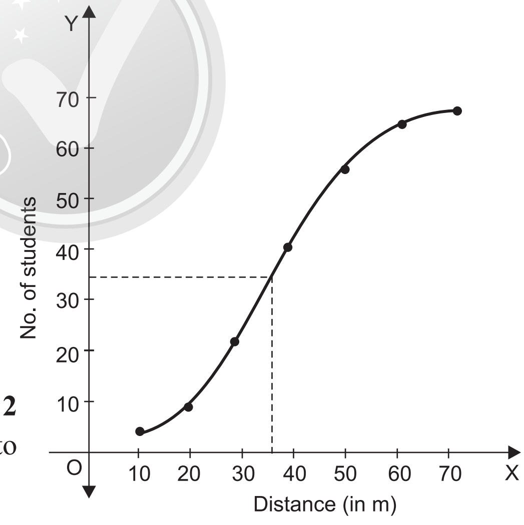
Daily pocket allowance (in ₹)	Number of children (f_i)	Mid-point (x_i)	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11–13	3	12	– 3	– 9
13–15	6	14	– 2	– 12
15–17	9	16	– 1	– 9
17–19	13	18	0	0
19–21	k	20	1	k
21–23	5	22	2	10
23–25	4	24	3	12
$\Sigma f_i = 40 + k$				$\Sigma f_i u_i = k - 8$

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$ **1**

$$18 = 18 + 2\left(\frac{k-8}{40+k}\right) \Rightarrow k = 8$$ **1**

OR

Less than	Number of Students
10	4
20	9
30	22
40	42
50	56
60	64
70	68



Less than Ogive

2

Median distance is value of x that corresponds to

Cumulative frequency $\frac{N}{2} = \frac{68}{2} = 34$

Therefore, Median distance = 36 m **1**